No.

Date

京大2013入試問題$^{\text{物理}}$ No.2 (電磁気学) 解答 (1)

II (1) イ、 $W = \dfrac{q^2}{2C}$, $C = \varepsilon_0 \dfrac{S}{d}$

$$\therefore W = \boxed{\dfrac{q^2 d}{2\varepsilon_0 S}}$$

ロ、 $W' = \boxed{\dfrac{q^2(d+\Delta d)}{2\varepsilon_0 S}}$

ハ、 $F = \dfrac{\Delta W}{\Delta d} = \dfrac{W'-W}{\Delta d} = \dfrac{q^2}{\Delta d \cdot 2\varepsilon_0 S}(d+\Delta d - d)$

$$= \boxed{\dfrac{q^2}{2\varepsilon_0 S}}$$

(2) ニ. $C_1 = \dfrac{\varepsilon_0 S_1}{d}$, $C_2 = \dfrac{\varepsilon_0 S_2}{d}$

直列: $V = \dfrac{q'}{C_1} + \dfrac{q'}{C_2} = \dfrac{q'd}{\varepsilon_0}\left(\dfrac{1}{S_1} + \dfrac{1}{S_2}\right)$

$$= \frac{(S_1 + S_2)q'd}{\varepsilon_0 S_1 S_2}$$

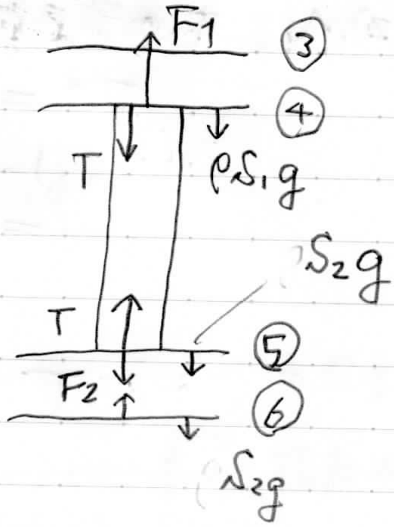
$$\therefore q' = \frac{\varepsilon_0 S_1 S_2 V}{(S_1 + S_2)d}$$

ハ.の結果より、 $F_1 = \dfrac{q'^2}{2\varepsilon_0 S_1} = \dfrac{1}{2\varepsilon_0 S_1} \cdot \dfrac{\varepsilon_0^2 S_1^2 S_2^2 V^2}{(S_1+S_2)^2 d^2}$

$$= \boxed{\dfrac{\varepsilon_0 S_1 S_2^2 V^2}{2(S_1+S_2)^2 d^2}}$$

京大2013入試物理問題Ⅱ（電磁気学）ー解答ー(2)

(2) ホ.



極板 ④, ⑤, ⑥ の力の釣合いより

$$\begin{cases} F_1 = \rho S_1 g + T & \text{—(a)} \\ T = F_2 + \rho S_2 g & \text{—(b)} \\ F_2 = \rho S_2 g & \text{—(c)} \end{cases}$$

(a) −(b) +(c) より

$$F_1 = (S_1 + 2S_2)\rho g$$

ニ. を代入し, $\dfrac{\varepsilon_0 S_1 S_2^2 V^2}{2(S_1+S_2)^2 d^2} = \rho(S_1 + 2S_2)g$ —(d)

ハ. の結果より F_1 と同様に $F_2 = \dfrac{q'^2}{2\varepsilon_0 S_2} = \dfrac{\varepsilon_0 S_1^2 S_2 V}{2(S_1+S_2)^2 d^2}$

(c)に代入 ∴ $\dfrac{\varepsilon_0 S_1^2 S_2 V^2}{2(S_1+S_2)^2 d^2} = \rho S_2 g$ —(e)

(d)/(e)

$$\frac{S_1 S_2^2}{S_1^2 S_2} = \frac{S_1 + 2S_1}{S_2} \quad \therefore S_2^2 = S_1^2 + 2S_2 S_1$$

$$S_1^2 + 2S_2 S_1 - S_2^2 = 0 \quad \therefore S_1 = (-1 \pm \sqrt{2})S_2$$

$S_1 > 0$, $S_2 > 0$ より

$$S_1 = (\sqrt{2} - 1)S_2 \quad \therefore \frac{S_1}{S_2} = \boxed{\sqrt{2} - 1} \quad \text{—(f)}$$

解答-(3)

(2) へ. ホの計算過程(e), 結果ホ, より

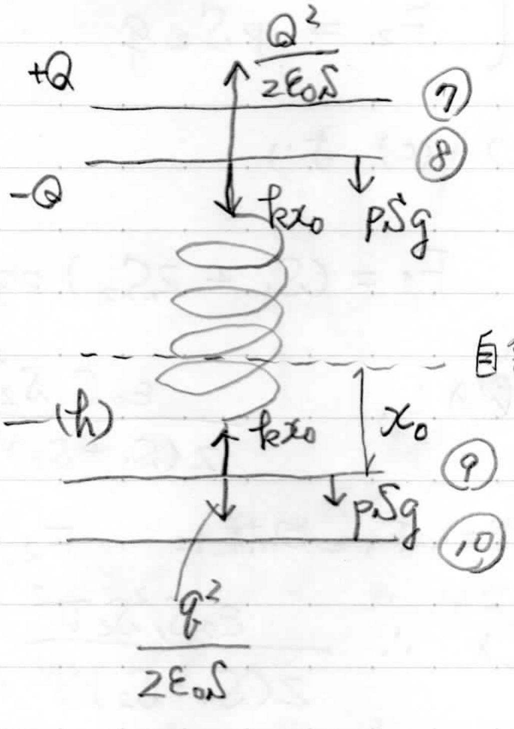
$$V^2 = \frac{2(S_1+S_2)^2 d^2 \rho g}{\varepsilon_0 S_1^2} = \frac{2(\sqrt{2})^2 S_2^2 d^2 \rho g}{\varepsilon_0 (\sqrt{2}-1)^2 S_2^2}$$

$$= \frac{4 d^2 \rho g}{\varepsilon_0 (\sqrt{2}-1)^2} = \frac{4(\sqrt{2}+1)^2 d^2 \rho g}{\varepsilon_0}$$

$$\therefore V = \boxed{2(\sqrt{2}+1) d \sqrt{\frac{\rho g}{\varepsilon_0}}} \quad \text{—(g)}$$

(3) ト.

極板⑨の力の釣合いより



$$kx_0 = \frac{q^2}{2\varepsilon_0 S} + \rho S g$$

$$\therefore x_0 = \boxed{\frac{1}{k}\left(\frac{q^2}{2\varepsilon_0 S} + \rho S g\right)} \quad \text{—(h)}$$

チ. スイッチを閉じると, ⑨, ⑩の電荷は0になる.

振動中心での力の釣合いより,

$$kx_0' = \rho S g \quad \therefore x_0' = \frac{\rho S g}{k} \quad \text{—(i')}$$

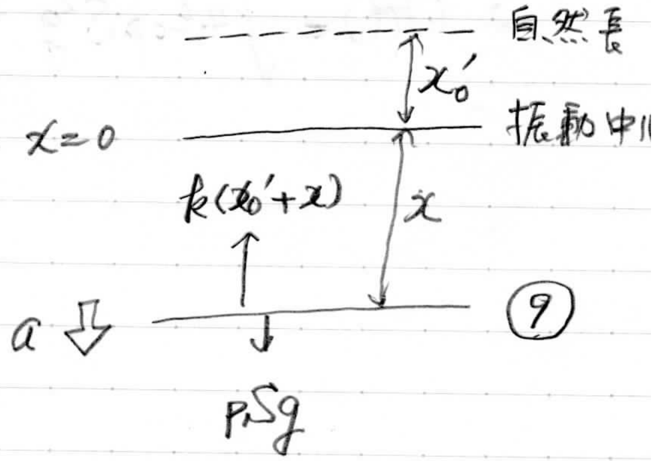
$$x_0 - x_0' = \boxed{\frac{q^2}{2k\varepsilon_0 S}} \quad \text{—(i)}$$

解答-(4)

(3) り. 振動を始めた位置が $v=0$, 振巾最大なので

$$A = x_0 - x_0' = \boxed{\frac{q^2}{2k\varepsilon_0 S}} \quad —(j)$$

ヌ.



極板⑨の運動方程式は

$$\rho S\cdot a = \rho S g - k(x_0'+x)$$

$$= -kx$$

$$\therefore\ a = -\frac{k}{\rho S}x$$

$$\therefore\ \omega = \sqrt{\frac{k}{\rho S}} \quad —(k') \qquad T = 2\pi/\omega = \boxed{2\pi\sqrt{\frac{\rho S}{k}}} \quad —(k)$$

問1.

$$x = A\cos\omega t \quad —(l)$$

⑧の力の釣合いより

$$\frac{Q^2}{2\varepsilon_0 S} = k(x_0'+x) + \rho S g$$

(i'),(l),(j),(k')を代入. $\frac{Q^2}{2\varepsilon_0 S} = \rho S g + \frac{q^2}{2\varepsilon_0 S}\cos\sqrt{\frac{k}{\rho S}}\,t + \rho S g$

$$= 2\rho S g + \frac{q^2}{2\varepsilon_0 S}\cos\sqrt{\frac{k}{\rho S}}\,t$$

条件は右辺の最小値 $2\rho S g - \frac{q^2}{2\varepsilon_0 S} \geqq 0$

$$\therefore\ q \leqq \boxed{2S\sqrt{\varepsilon_0 \rho g}} \quad —(m)$$

解答 -(5)

問2、　問1より

$$\frac{Q(t)^2}{2\varepsilon_0 S} = 2\rho Sg + \frac{q^2}{2\varepsilon_0 S}\cos\sqrt{\frac{k}{\rho S}}\,t$$

$$\therefore Q(t) = \sqrt{4\varepsilon_0 \rho S^2 g + q^2 \cos\sqrt{\frac{k}{\rho S}}\,t} \quad //$$